A constant force generator for the demonstration of Newton's second law

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A constant force generator is very helpful in demonstrating Newton's second law in the classroom. A device is described which avoids the disadvantages of setups currently used. It is based upon the induction of eddy currents. It is easy to handle and can be self-made.

I. INTRODUCTION

A device which generates an adjustable but constant force would be very useful in demonstrating Newton's second law on an air track. In principle, such a device could simply be a box with some mechanism inside which would pull with a constant force on a string leading out from the box (Fig. 1). "Constant" means that the force is independent of (1) the position of the end of the string, (2) the speed with which the string moves, and (3) the acceleration of the string. As far as we know, no such device is available on the market. Furthermore, we were not able to find any references to the construction of a similar device in the recent literature (since the invention of the air track in the early sixties).

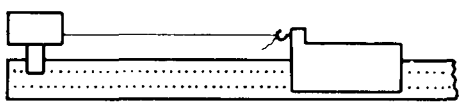

Fig. 1. In principle, a constant force generator is a box fixed at one end of the air track. A mechanism inside the box pulls with constant force on a string leading out from the box. The force is independent of the position, the speed, and the acceleration of the end of the string.

One could imagine several force generators which fulfill one or two but not all three of the above conditions. For example, a clock–spring that drives a drum on which the string is rolled up within the box (Fig. 2). Here the force does not depend upon the speed, and only weakly depends upon the acceleration (because of the inertia of the pulley and the spring). However, because the spring obeys Hooke's law, the force depends strongly upon the position of the end of the string.

Another force generator is shown in Fig. 3. It is current-

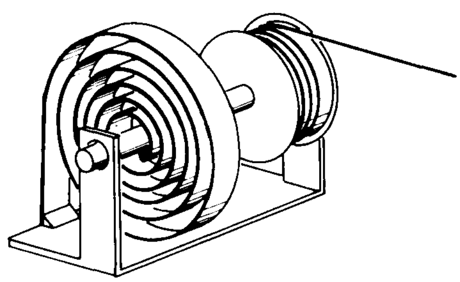

Fig. 2. This clock–spring device generates a force which is independent of speed and acceleration, but which strongly depends upon the position of the end of the string.

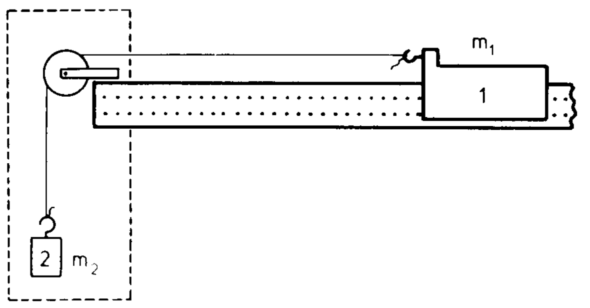

Fig. 3. Device currently used to generate a constant force. The force exerted on body 1 alone strongly depends upon acceleration. Only the force on the "compound" system 1 + 2 is constant.

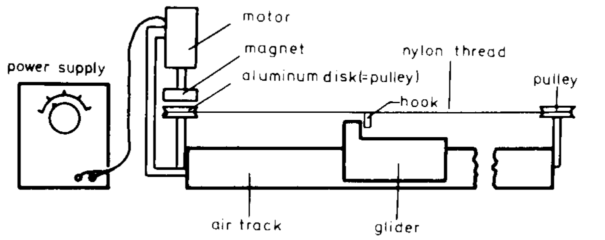

Fig. 4. General view of a constant force generator. The rotating magnet induces eddy currents in the aluminum disk, and thus exerts a couple on the disk.

ly used to demonstrate Newton's second law but, like the first example, it also represents no more than a compromise.[1] The generator consists of body 2 and the pulley (inside the dotted line). Body 1 is the system to which Newton's second law will be applied. The force which acts via the string on body 1 is independent of the position of body 1 (and thus of the string and body 2). Furthermore, it is independent of the speed of body 1 (and that of the string and body 2). However, the force depends strongly upon the acceleration a of the two bodies: the force exerted by body 2 on the string is $m_2(g - a)$.

It is hard to measure the force in this experimental setup because this measurement must take place while the system is accelerating. The force can't be determined in a static measurement before the acceleration experiment begins. On the other hand, the force can't be calculated in the context of the curriculum because this calculation presupposes the law which is to be demonstrated in the experiment in the first place. One usually gets around this dilemma in the following way: Newton's second law isn't demonstrated for body 1 alone, but for the "compound" body 1 + 2, with mass $m_1 + m_2$. Now, body 2 is not only part of the force generator. It is also part of the system under investigation. The total force exerted on the new system, $F = m_2 g$, depends only upon the mass of body 2 and no longer upon the acceleration.

This way of proceeding may calm the teacher's conscience, but, unfortunately, it mixes the two properties of the mass of the investigated body—inertial and gravitational—in a puzzling way. This makes matters even worse since the original purpose of the experiment was to demonstrate the effect of inertia alone.

II. DESCRIPTION OF THE CONSTANT FORCE GENERATOR

We now describe a device which generates a force independent of position, speed, and acceleration of the string to within a very good approximation. The device is simple and robust and can be easily constructed (Figs. 4 and 5). An electric motor is mounted vertically at one end of the air track. This motor turns a permanent magnet, which is attached to the shaft of the motor. The poles of the magnet are all on the lower side, so that the lines of magnetic flux loop "horseshoe fashioned" through an aluminum disk mounted on a separate axle a small distance under the magnet. The magnetic field lines induce eddy currents in the disk and in this way couple the magnet to the disk. The disk has a groove on its perimeter. Thus it can act as a pulley. It pulls on one leg of a nylon loop which wraps around a second pulley at the other end of the air track, and then back again to the first pulley. A hook is attached somewhere along this thread. This hook can be used to puli a glider.

The motor is a small, high-speed (12 000 rpm), 10-W motor. The ceramic disk magnet has a diameter of about 3 cm. It may have two or more poles all on the same side. The pulleys, about 4 cm in diameter, are very light (10 g each). They are supported by ball bearings.

The force exerted on the thread can be varied in two ways: (1) by changing the rotational speed of the motor (by means of its power supply) and (2) by changing the distance between the magnet and the aluminum disk (from about 1/2 mm to several mm).

The force is independent of the state of motion of the thread. Thus the force can easily be set to any desired value before the beginning of the acceleration experiment: you simply attach a spring balance to the hook and turn the knob of the power supply of the motor until the balance indicates the desired value. In this way the force is set to within the accuracy of the spring balance, i.e., to within about 5% of the maximum deviation of the balance.

Why, now, does the device generate a constant force? First, it is obvious that the couple exerted on the disk, i.e., the force exerted on the thread, is independent of the position of the hook.

Second, the force is independent of the speed as long as the angular velocity of the aluminum disk remains small compared to that of the magnet. This can be seen from the following argument. The couple exerted by the magnet on the disk depends upon the intensity of the eddy currents induced by the magnet, whatever their distribution in the disk might be. According to Faraday's law these induced currents are proportional to the difference of the angular velocities $\Delta\omega = \omega_m - \omega_d$ between the magnet m and the

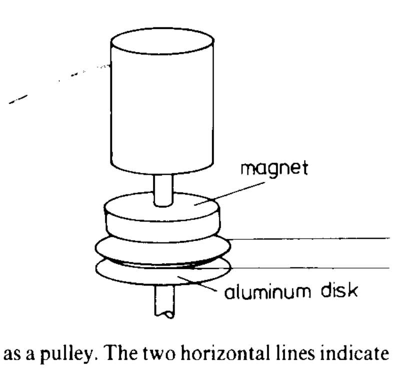

Fig. 5. Disk acts as a pulley. The two horizontal lines indicate the two legs of a loop of thread.

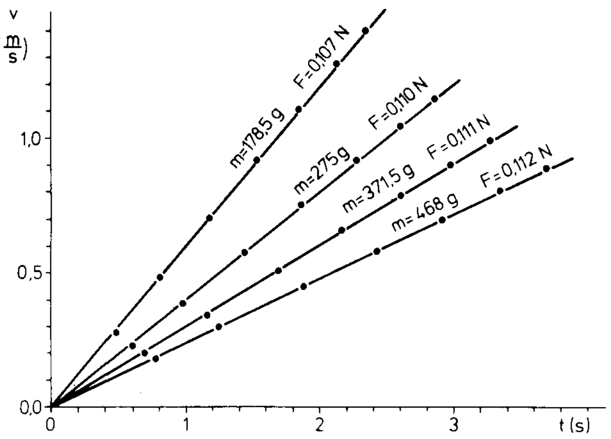

Fig. 6. Velocity versus time curves for four gliders of different mass. The measurements are shown as points. The straight lines are a good fit through the points. From the slope of the lines and the mass of the corresponding glider, the accelerating force has been calculated (shown next to each line).

disk d. Because of the high rotational speed of the motor, the angular velocity of the magnet is always very large ($\sim 1000\ \mathrm{s}^{-1}$) compared to that of the disk (maximum 80 s^{-1}). Therefore, we have

$$\Delta\omega \simeq \omega_m ,$$

i.e., $\Delta\omega$ is nearly independent of ω_d. To within the same approximation, the couple exerted on the disk is independent of ω_d.

Third, the force is independent of the acceleration as long as the inertia of the two pulleys is small compared to the inertia of the system under study. This condition can be written as

$$J \ll mr^2,$$

where J is the sum of the moments of inertia of the two pulleys, m is the mass of the glider to be accelerated, and r is the radius of the pulley. The moment of inertia of the pulleys is about 40 g cm^2. With $r = 2$ cm and a typical glider mass of 400 g, mr^2 is 1600 g cm^2. Thus mr^2 is a factor of 40 greater than J, which satisfies the quoted condition.

Figure 6 shows how well the device works. Four air-track gliders with masses differing by more than a factor of 2 were accelerated all with the same force setting (to within approximately 5%). The speed of the gliders was measured as a function of time. The points obtained in each case all fall on a straight line to within a good approximation. The slope of these curves is equal to the acceleration. Thus it can be seen that the acceleration of each glider is independent of its position and speed. If the product of the mass of a glider with its acceleration (= the slope in Fig. 6) is calculated, the same value (about 0.11 N) is obtained to within a good approximation in all cases. Furthermore, this value is in agreement with the force measured statically. It follows that the force exerted on a glider is independent of the mass of the glider and thus of its acceleration.

III. FURTHER APPLICATIONS OF THE DEVICE

In addition to the demonstration of Newton's second law, other experiments can be carried out with this device. Here are two examples:

(a) In this experiment, the hook on the thread pushes the glider "from behind." Whenever the thread is stopped, the glider continues to move at constant speed. It is now possible to exert a constant force on the glider for a limited interval of time. Initially, let the hook touch the resting glider while holding the thread (or one of the pulleys) still with one hand. Then release the thread (or the pulley) for the desired interval of time before stopping it again. This releasing and holding could also be done with a simple electromechanical device which is triggered by a clock.

If the momentum $\mathbf{p}$ of the glider is measured after the force is switched off, the relation

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

can be verified.

This experiment can also be used to transfer momentum of any desired value to a glider, without knowing the gliders' mass.

(b) If the hook is attached to the glider, the glider behaves like a body in a horizontal homogeneous gravitational field. The field strength of this "field" can be chosen at will. Thus free-fall can be simulated in slow motion, for example, as is the case for a falling body at the surface of the moon. A very nice experiment can be carried out when a spring bumper is attached at one side of the glider. The glider can then collide against a stop at one end of the air track. If the glider now "falls" against this stop, it bounces back, falls again and so on, just like a rubber ball bouncing on the floor—but all this can be made to happen very slowly. One sees what it would be like to bounce a ball on the surface of the moon.

[1] *Physics Demonstration Experiments*, edited by Harry F. Meiners (Ronald, New York, 1970), p. 255.