Subject:
Physics students learn that for an induced electric field the electric potential is not defined: “The existence of rotational electric fields shows that not every field has an electric potential […] Such fields have closed field lines. An electric charge can gain any amount of energy when moving on a closed path.”

Deficiencies:
We limit ourselves to consider electric fields. Similar arguments hold for magnetic fields and also for velocity fields of flowing liquids.

Among the electric fields there are two classes with particular properties: Conservative fields and rotational fields. A conservative electric field is a field for which
\[ \nabla \times \mathbf{E} = 0 \]
everywhere. That means that the field must have sources somewhere, i.e. \( \nabla \cdot \mathbf{E} \) cannot be zero everywhere. Otherwise there would not be a field at all.

The places where \( \nabla \cdot \mathbf{E} \neq 0 \) are sometimes called flux sources.

A pure rotational field is a field for which
\[ \nabla \cdot \mathbf{E} = 0 \]
everywhere and
\[ \nabla \times \mathbf{E} \neq 0 \]
somewhere.

We call the places where \( \nabla \times \mathbf{E} \neq 0 \) rotation sources.

In general, a field will have both kinds of sources and thus will not belong neither to the one nor to the other category.

Nevertheless, these concepts play an important role in the teaching of electrodynamics. The reason is that one often imagines that there is nothing else in the world than an electric dipole, a plate capacitor or a current-carrying solenoid. About their field simple statements can be made as for instance the following: The electric field of an electric dipole is a conservative field, or the electric field around a solenoid whose electric current is changing in time is a rotational field.

The simplicity of this classification sometimes gives rise to a conclusion that overshoots the target. An example is our citation: An induced electric field has no electric potential.

In order to see the problem we first have to obtain clarity about how we want to employ the word “field”.

Sometimes we speak of the electric field of a point charge, of a dipole or a capacitor (or of the magnetic field of a solenoid, a current loop or a bar...
magnet). When doing so we imagine that there is nothing else in the world than this point charge, dipole etc.

In other occasions we speak of the field in a given domain of space and it may be that the sources of this field are not our primary concern.

Statements as that of our citation refer to the first of these situations. They are global and general statements and they refer to systems of infinite extension. They are reasonable and useful when the intention is to get certain general insights about electrodynamics, but sometimes they are inappropriate. When dealing with a practical problem we are not interested in a statement about the world at large, but only about a given region of space. So a practical question may be: Does the region of space that we consider contain any flux or rotation sources? If there are no rotation sources within this region, we can define a potential. If the curl of the field is non-zero only at certain places within the considered space then we can cut out a simply-connected region that does not contain curl sources and define a potential for this region. Whether there are rotation sources outside of our region does not have any importance for our decision in favor or against a potential. If we took the statement of our citation at face value, one could never employ the useful tool “potential”. It would be forbidden to say that the neutral conductor of the power grid is at zero potential since for the total field distribution of the circuit somewhere in a transformer we have

$$\nabla \times \vec{E} \neq 0.$$ 

Or consider an electronic device: When it runs on battery, a potential field would exist, but when it is connected to the mains, there would be none.

*Origin:*

In electrodynamics we like to operate with simple systems like point charges, dipoles, solenoids etc.

*Disposal:*

Who believes that a rule is important may proceed as in [1]:

“Outside of the conductor a multivalued magnetic potential exists; for the calculation of the field this ambiguity does not play any role.”…“Inside of the conductor there is no magnetic potential.”

Who estimates that this approach is too fussy, will not give so much importance to the subject. In particular he will not formulate a statement like: “For an induced electric field a potential cannot be defined.” Whether a description with a potential is possible will be decided with regard to the space domain that is relevant for the particular problem.


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