Subject:
“Second law: The acceleration $a$ of a body is directly proportional to the force $F$ acting on the body and inversely proportional to the mass $m$ of the body, i.e.;

$$F = m \cdot a.$$  

“The magnitude of the centripetal force on an object of mass $m$ moving at tangential speed $v$ along a path with radius of curvature $r$ is:

$$F = ma_c = \frac{mv^2}{r}.$$”

Deficiencies:
Clearly there is no objection to the validity and to the usefulness of the equation

$$F = m \cdot a.$$ \hspace{1cm} (1)

We believe, however, that it is not convenient to call it “Second law”, or “Fundamental law of motion”, since it subsumes two other law that should be clearly kept apart from one another. The first one is indeed Newton’s second law

$$F = \frac{dp}{dt}.$$ \hspace{1cm} (2)

It tells us that momentum can change only when a momentum current is entering or leaving a system (in other words: when “a force is acting on the system”). It thus claims that momentum is a conserved quantity.

In order to obtain equation (1) we still need the equation

$$p = m \cdot v,$$ \hspace{1cm} (3)

or its time derivative

$$\frac{dp}{dt} = m \frac{dv}{dt}.$$

The character of equation (3) is different from that of equation (2). It is what in another context one would call a constitutive equation. Such equations are valid for certain systems under certain circumstances. So, the equation

$$p = m \cdot v$$

is valid only as long as the velocity is small when compared with $c$, and it is not valid for the electromagnetic field, since this system is not described by the variables $m$ and $v$.

In order to make our argument clearer consider the corresponding electric laws. The following analogy between laws of mechanics and electricity is well-known:
mechanics

\[ F = \frac{dp}{dt} \]

\[ I = \frac{dQ}{dt} \]

\[ p = m \cdot v \]

\[ Q = C \cdot U \]

\[ F = m \cdot \frac{dv}{dt} \]

\[ I = C \cdot \frac{dU}{dt} \]

It is obvious, that to the equation

\[ I = C \cdot \frac{dU}{dt} \]

one would not give a name like “fundamental law of electricity”.

When skipping equation (2) and declaring that equation (1) is the fundamental law, or also the Second law, then there is no need to mention momentum. Apparently, this is considered an advantage. Indeed, in the usual course of teaching mechanics the quantity force, i.e. momentum current, is discussed extensively at the beginning, whereas momentum has to wait until collision processes are discussed.

The tendency to circumvent momentum can also be observed at other instances, see our second citation. Here again the intermediate result is missing. The first step to get the centripetal force is to calculate the time rate of change of momentum of the rotating body:

\[ \frac{dp}{dt} = m \frac{v^2}{r} . \]

The term on the right hand side of the equation only contains quantities that characterize the rotating body. Only by using Newton’s second law we get:

\[ F_c = m \frac{v^2}{r} . \]

**Origin:**

The disregard of momentum and its reduction to a mere invariant in collision processes is not a relict of the early times of mechanics. On the contrary, already before Newton’s time and at Newton’s time momentum was that quantity in whose balances one was interested. Its original latin name was *quantitas motus*, which can be translated as *quantity of motion* or also as *amount of motion*. Momentum gained importance in modern physics. Relativistic physics tells us that momentum density is a component of the energy-momentum tensor, and thus belongs to the sources of the gravitational field. Therefore, the apparent disregard of momentum in elementary mechanics seems incomprehensible.

We believe that we have to blame Leibniz. In the famous controversy about the “true measure of force” between Leibniz and the Cartesians the question was which of the two expressions \( m \cdot v \) and \( m \cdot v^2 \) is the “correct” measure of the amount of motion. Today we know that physics needs both expressions. The first one is what we call today momentum and the second (apart from a factor of 2) is the kinetic energy. Both of them correspond to what at Leibniz’s time was called force by some, and what today we would call impetus or drive or momentum (in the colloquial sense of the term).

Now, in the teaching tradition of mechanics, this everyday concept of impetus was primarily associated with the physical quantity kinetic energy, i.e. essentially with Leibniz’s \( m \cdot v^2 \). The simple reason is that we generally introduce kinetic energy before momentum. Thus, when momentum is introduced the place for a physical quantity that measures what we intuitively associate with the concept of impetus is already occupied. As a result, most
students consider momentum as the more abstract quantity. They learn that momentum measures something rather similar to kinetic energy without being the same quantity. Thus momentum is perceived as a more difficult quantity and its role is essentially reduced to an invariant in collision processes.

By the way: In this respect the fate of momentum is somewhat similar to that of entropy. Initially this quantity was a perfect measure of what in colloquial term would be called heat. After the introduction of energy and the discovery of its conservation the name and the mental picture that it associated with the word heat was transferred to the differential form $dQ$. As a result entropy was now considered a rather “abstract” quantity.

Disposal:

Introduce momentum as a basic quantity right at the beginning of mechanics. Introduce it as a measure of what in colloquial terms would be called impetus.

Introduce Newton’s second law as Newton did it: $F = \frac{dp}{dt}$. Introduce the relation $p = m \cdot v$ later, in the same way as you discuss the relation $Q = C \cdot U$ after the introduction of the electric charge $Q$.

Regarding the circular motion, before introducing the centripetal force show that

$$\frac{dp}{dt} = m \frac{v^2}{r}.$$

Friedrich Herrmann, Karlsruhe Institute of Technology