Subject:

The law of conservation of angular momentum is often introduced as follows: We consider a mass point. We write the cross product of the vector on both sides of Newton’s second law

\[ F = \frac{dp}{dt} \]

and the position vector \( r \) (relative to an arbitrarily chosen origin). We get a relation between the torque and the time rate of change of the angular momentum:

\[ M = \frac{dL}{dt} \]

We write the corresponding expression for two or more mass points and take into account that for the internal interaction forces there is

\[ F_{ik} = - F_{ki} \]

and that these forces are parallel to \( r_i - r_k \). We then find that the time derivative of the angular momentum of the system of mass points is equal to the sum of the torques of the external forces. From this follows the law of angular momentum conservation: “The angular momentum of a system remains constant, if no external torque acts on the system.”

Deficiencies:

Our foregoing derivation of the angular momentum conservation is somewhat short, since we believe that it is known to the reader. In a text book it easily needs an entire page with about 10 lines of equations. It is not hard to follow such a derivation step by step, and at the end, the student will probably be convinced that the law of momentum conservation must be valid. However if we ask the student, what actually has been proven on this page, he or she might run into trouble. The derivation starts with Newton’s second law, which is equivalent to the law of momentum conservation, and the result of the calculation is the law of angular momentum conservation. It is unavoidable that the student believes, angular momentum conservation has been mathematically derived from momentum conservation. It is needless to say that this is not true. There will hardly be a student who understands the trick that was employed. They will even not suspect that there was a trick. Actually, in the above derivation angular momentum conservation is not derived from momentum conservation, but angular momentum conservation is fed into the calculus when saying that the forces \( F_{ik} \) and \( F_{ki} \) are parallel to \( r_i - r_k \).

Fig. 1 shown something that does not exist in reality. Two bodies exert forces on one another, that are equal and of opposite direction (\( F_{12} = - F_{21} \)), but which are not parallel to \( r_i - r_k \). They obey Newton’s third law and thus mo-
mentum conservation, but since they are equivalent to a torque, the angular momentum of the system should increase, and it would do so without an external torque. But there are no such forces. They are forbidden by the law of angular momentum conservation. Thus the claim that $F_w$ and $F_k$ are parallel to $r_i - r_k$ is equivalent to the claim that angular momentum is a conserved quantity.

In summary, a somewhat lengthy calculation is carried out, into which angular momentum conservation is injected, and at the end, one is happy that the law of angular momentum conservation comes out. But why then the calculation?

*Origin:*

Newton's laws contain not more and not less than momentum conservation. Due to their great success the idea has spread that they are more than just a simple conservation law. They seem to be the be-all and the end-all of physics, the basis from which everything else can be derived. Sometimes, even energy conservation is derived from Newton’s laws — again with a trick.

*Disposal:*

Introduce angular momentum as a quantity of its own right, for which a conservation law is valid. This does not exclude to show how the angular momentum of a system of mass points is related to the momenta of its constituents.

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