Subject:
To represent an electric, a magnetic or a gravitational field graphically commonly field line pictures are drawn. In the case of the electric and the gravitational field sometimes equipotential surfaces are also represented.

A field line picture expresses two aspects of the field:

1. It shows the direction of the field strength vector in each point of the field. The field strength vectors point in the direction of the tangents of the field lines.
2. It tells us where the “sources” of the field are located. These are the places where the field lines begin or end.

Sometimes it is said that a field line picture allows to read the magnitudes of the field strength vectors. Actually this is true only in special cases [1,2].

Deficiencies:
A graphical representation of a field allows us to grasp at a single glance, what would be complicated to express in words. (“A picture is worth a thousand words.”) However, although there are several possibilities to graphically represent a field mostly a single method is used: the field line picture. We are so accustomed to this representation that it hardly comes to our mind, that there are alternatives. One such alternative are the surfaces that are orthogonal to the field lines, the field surfaces.

Fields are often introduced as rather abstract entities. Therefore for many students the field lines are the straw which they will catch. And the result is often that they identify the field lines with the field.

Origin:
For Maxwell it was natural to represent all fields by both the field lines (“lines of force”) and the field surfaces (“equipotential surfaces”), Fig. 1. This was a method to realize a suggestive picture of an invisible object. At the turn of the century serious doubts about the existence of an aether came up, and the aether was banned from physics. As a consequence, the field degenerated into an abstract entity, hardly more than a mathematical concept for calculating forces. From now on field lines were no more than auxiliary lines that represented the direction of the force on a test particle. The orthogonal surfaces survived only in the form of equipotential surfaces in the special case that the fields were conservative. Since a potential can only be defined for a conservative field, the opinion was now, that drawing the orthogonal surfaces makes sense only in such fields. Apparently, it was not noticed that the only problem was the name. Actually orthogonal surfaces can also be drawn for nonconservative fields. Then they are not equipotential but they are just as useful as the equipotential surfaces in conservative fields. Actually, they become particularly interesting for non-conservative fields, since they indicate clearly where the curl of a field is located.
Disposal:

In the following we call flux sources those places within a field where the divergence is different from zero. We call circulation sources the places where the curl of a field is different from zero.

Just as flux sources are places where field lines begin or end, circulation sources are places where field surfaces terminate. In a field line picture the flux sources are easily seen, whereas the field surfaces indicate clearly the circulation sources. That is why one best represents both in each field picture: field lines and field surfaces (in a two-dimensional plot the field surfaces also appear as lines).

Consider as an example electric fields. The flux sources are electric charges, the circulation sources are places where the magnetic flux is changing with time.

Fig. 2 shows two linear charges (thin charged wires, perpendicular to the drawing plane) and three thin “linear” coils whose magnetic flux is changing with time. The coils are also perpendicular to the drawing plane, and therefore appear as points.

The Figure can also be interpreted as a magnetic field. Then the flux sources are magnetic line charges (linear magnetic poles), the circulation sources are electric currents.


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Fig. 1. Superposition of the magnetic field of an electric conductor (perpendicular to the drawing plane) and a homogeneous magnetic field from Maxwell’s Treatise on Electricity and Magnetism

Fig. 2. Field of two flux sources and three circulation sources (field lines: black; field surfaces: grey)