Subject:
The Bernoulli equation

\[ p + \rho \cdot g \cdot h + \frac{\rho \cdot v^2}{2} = \text{const} \]

holds for a stationary, incompressible, inviscid (frictionless) fluid. \( p \) is the pressure, \( \rho \) the mass density, \( g \) the gravitational field strength, \( h \) the height (positive direction upwards) and \( v \) the velocity. Here, \( \text{const} \) means that the sum at the left hand side of the equation does not change when one is moving along a streamline. If the values of the local variables are the same in every point of a cross section, then the condition is even less restrictive. Then \( \text{const} \) means “has the same value at every cross sectional area”.

Usually, the equation is interpreted in the following way. There exist several types of pressures: the static pressure \( p \), the gravitational pressure \( \rho \cdot g \cdot h \) and the dynamic pressure or stagnation pressure \( (\rho/2) \cdot v^2 \). The Bernoulli equation tells us that the sum of these three pressures is constant (under the conditions that have been mentioned).

Deficiencies:
Qualitatively and put into words, the equation states the following:

1. At places where the fluid is rapid, pressure is lower than where it is slow.
2. Pressure increases when going down within the fluid.

These statements contain only one pressure, which is the quantity \( p \) in the Bernoulli equation. Both terms \( \rho \cdot g \cdot h \) and \( (\rho/2) \cdot v^2 \) have the dimension of a pressure, but they are not what we normally understand by a pressure. The terms of a sum do not necessarily represent physical quantities of the same kind. The term \( \rho \cdot g \cdot h \) cannot be the gravitational pressure, since the gravitational pressure increases when going downwards whereas \( \rho \cdot g \cdot h \) decreases.

Origin:
Probably the objectionable interpretation is due to the desire to consider pressure as a quantity for which a kind of conservation law is valid. Indeed, the formulation “The total pressure is constant” reminds a certain way of expressing the conservation or energy, electric charge or angular momentum: “In an isolated system the total amount of energy (electric charge, angular momentum) remains constant.” Such statements are elegant, since it is easy to formulate them and they are universally valid. Thanks to the Bernoulli equation also pressure could enter the illustrious circle of the conserved quantities. Moreover, such a conclusion seems natural, since Bernoulli’s equation can be derived from the energy conservation law.

We believe that arguing in this way is exaggerating. Pressure cannot be a “conserved quantity”, since a necessary condition for being conserved is that the quantity is extensive – which is not true for the pressure.
One might object that, giving the name “dynamical pressure” to the term \((\rho/2) \cdot v^2\) can be neither false nor true, since it is only a question of giving a name. However, the choice of a name can be more or less appropriate and we believe, that the name pressure for the terms \(\rho \cdot g \cdot h\) and \((\rho/2) \cdot v^2\) is not appropriate. Pressure is a quantity for which we have a sound intuition. Calling \((\rho/2) \cdot v^2\) a pressure would cause our students to believe that pressure is a difficult concept. The uplifting attribute “dynamic” further supports this idea.

Moreover, the expression \((\rho/2) \cdot v^2\) is known as the density of the kinetic energy. So one could argue to call all the terms of the Bernoulli equation energy densities: We might call \(p\) the static energy density, \((\rho/2) \cdot v^2\) the dynamic energy density and \(\rho \cdot g \cdot h\) the gravitational energy density. It is obvious that this would not be a good idea.

**Disposal:**

Read the Bernoulli equation as follows: The pressure decreases when (1) the velocity increases, (2) height increases. Both statements are plausible.

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