Subject:
When studying electrodynamics at the secondary school Lenz’s law is introduced. According to this law the magnetic field of an induced current opposes the original change in magnetic flux. Lenz’s law thus makes a claim about the direction of a vector: the current density vector of the induced current.

Moreover, one shows that the minus sign in Faraday’s law of induction

\[ U = -\frac{d\Phi}{dt} \]

is a consequence of Lenz’s law. The argument goes as follows: Introduce an iron core into a solenoid that is connected to a battery. It is observed that the current intensity decreases for a moment. Since \( \frac{d\Phi}{dt} \) is positive (so it is argued) and the induced electromotive force (emf) is negative, there must be a minus sign.

Lenz’s law is often interpreted in yet another context: Thanks to the minus sign electromagnetic processes obey the law of energy conservation.

Deficiencies:
On the one hand we have to do with a complicated meshwork of statements that got into the school curriculum by tradition, and on the other hand with mistakes in the line of argument.

1. When explaining an algebraic sign in an equation, it should be possible to verify this sign, by measuring the quantities that appear in the equation. The minus sign in the law of induction tells us, that the induced emf has the opposite sign as the time rate of change of the magnetic flux, i.e. \( \frac{d\Phi}{dt} \). In order to check this claim one must know how to measure both these quantities including the algebraic sign.

Let us imagine that the flux change is as follows: The \( B \) vector points in the positive \( x \) direction and its absolute value (and thus its \( x \) component) increases. What is its algebraic sign? Does this sign remain the same when the coordinate system it rotated by 180°, so that the \( B \) vector now points in the negative \( x \) direction? Now this flux is surrounded by a circular conductor that is placed in the \( y-z \) plane and closed by a voltmeter. How can we read the correct sign from the voltmeter?

Our pupils do not learn how to answer these questions. Therefore the effort in finding out that there should be a minus sign was in vain.

2. In some books we found a mistake in the derivation of the minus sign. This mistake seems to have a life of its own, since although the minus sign can be derived in various different ways, the pattern of arguments, together with the mistake can be found on and off. When the current in the solenoid is switched on, so it is said, \( \frac{d\Phi}{dt} \) is greater than zero. This conclusion is not correct. The magnetic flux depends depends on the flux density according to

\[ \Phi = \int B dA \]
Since \( dA \) is a vector, \( \phi \) turns out to be positive or negative depending on the orientation of the area of integration.

3. When paying so much attention to the minus sign in one equation, then the plus signs in many other equations should be worth a similar consideration. What does it mean that in Newton’s second law, in Ohm's law or in \( P = v \cdot F \) there is a plus sign? And which sign do we need to write in Hooke’s law? Let us consider a spring that is tended in the vertical direction. We can identify four forces, all related to the spring: the force exerted by the spring on the upper suspension, the force exerted by the spring on the lower suspension, the force exerted by the upper suspension on the spring and the force exerted by the lower suspension on the spring. All of these forces have the same absolute value, two of them are directed upwards, two downwards. So we can take our choice. Although the question of the sign in Hooke’s law is not easier to answer than that in the law of induction it is usually dismissed.

4. Why does one insist just in the context of the law of induction that an incorrect sign would mean a violation of the law of conservation of energy? The impression results that induction has something particular to offer in this respect. Actually, there are plenty of other physical laws that would violate the energy conservation law if a sign is inverted: \( U = R \cdot I \), \( P = v \cdot F \), \( F = -D \cdot s \ldots \) Moreover, by changing a sign arbitrarily any other conservation law can be violated.

5. We do not object to honor a scientist by attributing his name to a physical law. But isn’t it a little exaggerated to associate the orientation of a current density vector, which anyway follows from Maxwell's equations, to a scientist's name and to give the status of a physical law?

**Origin:**

Usually a comprehensive theory emerges from predecessors. Unfortunately we sometimes afford the extravagance of teaching not only the last, and in general simplest and clearest version of the theory, but also its predecessors. Lenz’s law is only one of many examples.

In 1834, three years after Faraday’s discovery of the electromagnetic induction, Lenz had formulated his rule in the following way: "If a constant current flows in the primary circuit A, and if, by the motion of A, or of the secondary circuit B, a current is induced in B, the direction of this induced current will be such that, by its electromagnetic action on A, it tends to oppose the relative motion of the circuits.." [1]

At that time the question for the direction of the induced current was not a trivial one and Lenz’s law was a new statement. Only 13 years later Helmholtz made an energy balance and showed that Lenz’s rule could also be obtained from energy conservation. Another 25 years later Maxwell published his comprehensive theory of electrodynamics in which Lenz's was absorbed.

The fact that we still today give so much attention to the minus sign in the law of induction is no more than a convention. We must not forget that when teaching, a problem often becomes only a problem if the teacher declares that it is a problem.
Disposal:

Regarding Lenz's law, we refrain from teaching it as a law. Instead we formulate two "hand rules", one for the right and one for the left hand. The right-hand rule is an expression of Maxwell's forth equation, the left-hand rule follows from his third equation.

Right-hand rule: Point the thumb of your right hand in the direction of an electric current. Then the curled fingers point in the direction of the magnetic field vector.

Left-hand rule: Point the thumb of your left hand in the direction of the change $dB$ of the magnetic flux density. Then the curled fingers point in the direction of the electric field vector of the induced electric field.

For the handling of the law of induction there are two possibilities:

Either one explains carefully how one can determine the sign of the various physical quantities, in particular the electric current intensity and the voltage. But that is not all: One also has to explain the convention that associates the algebraic sign of a path integral with that of a surface integral in Stokes' theorem: When the thumb of your right hand points in the direction of surface element of the integral your curled fingers point in the direction of the integration path of the path integral. Regarding the secondary school, we do not recommend to proceed in this way, since it is cumbersome and does not result in any important insight.

So we prefer the second solution: Do not hesitate to formulate the law of induction with only the absolute values of the pertinent quantities.


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