29 Momentum as the product of *m* and *v*

Subject:

Usually the momentum of a body is defined as the product of its mass and its velocity:

$$\boldsymbol{p} = m \cdot \boldsymbol{v} \,. \qquad (1)$$

Thus, $\boldsymbol{p}$ is nothing else than an abbreviation for the product of m and v. Momentum seems to be a typical example of a “derived quantity”. In some books momentum is explicitly called an auxiliary quantity [1].

Deficiencies:

There are several arguments to introduce momentum not as a derived but as a basic quantity in its own right.

1) Momentum is a conserved quantity. This property makes it easy to measure the momentum of a moving body without recourse to equation (1) [2,3]. Since (gravitational) mass and velocity can be measured independently, equation (1) can be verified experimentally.

2) Equation (1) does not hold for every system. The momentum of the electromagnetic field cannot be calculated with this equation. The momentum density of the electromagnetic field can be calculated form the electric and the magnetic field strength:

$$\rho_p = \frac{\boldsymbol{E} \times \boldsymbol{H}}{c^2}$$

3) There is a far reaching analogy between mechanics and electricity: a correspondence between physical quantities and relations between these quantities. For example, the electric analog of the conserved extensive quantity momentum is electric charge, and the analog of the intensive quantity velocity is the electric potential. The analog of equation (1), which tells us that for non-relativistic velocities momentum is proportional to the velocity, is the equation

$$Q = C \cdot U, \qquad (2)$$

which tells us, that for a capacitor with fixed plates, the charge is proportional to the potential difference between the plates. A comparison of equations (1) and (2) shows that mass can be interpreted as “momentum capacitance”. The greater the mass of a body is, the more momentum it contains at a given velocity.

The comparison shows that it is no convenient to define momentum by equation (1). This is as if one would define electric charge by equation (2), instead of introducing it as a quantity in its own right, which can be measured without recourse to U and C.

4) To introduce momentum directly as a self-contained quantity is also suggested by the fact, that momentum (or more exactly momentum density) is a component of the energy-momentum tensor. That means that for the gravitational field momentum plays a similar role as electric charge for the electromagnetic field. Together with the energy density, the energy flow

density and the momentum flow density, it belongs to the sources of the gravitational field. The sources of a field play an important part in the fundamental interactions, and it seems not convenient to consider them as derived quantities.

Origin:

In contrast to the electric charge, the physical quantity momentum came into existence in a long historical process. In the 17th century it was a professed aim of the mechanical sciences to formulate the laws that govern collision processes. It was correctly expected that an invariant quantity should play a decisive role and it was tried to express this quantity as a combination of mass and velocity.

In 1644 Descartes published his Principia philosophiae, in which he claimed the conservation of the product of mass and velocity, the *quantitas motus*, the amount of motion. Some decades later Leibniz believed to prove that the product of the mass and the square of the velocity is the "correct" invariant in a collision process. As a consequence the famous, long-lasting dispute about which is the true "measure of force" broke out, which was brought to an end only in 1726 by Daniel Bernoulli, and in which there were no winners and no losers. What happened was the emergence of two quantities, one of which is what we now call momentum and the other kinetic energy.

Naturally, the result was a momentum that was *defined* by equation (1). Only much later it was discovered that if a conserved quantity momentum is to be constructed relation (1) has to be abandoned. The Theory of special relativity tell us that the new, conserved momentum is not proportional to velocity. Equation (1) was saved by introducing a velocity-dependent mass.

Disposal:

Introduce momentum as a quantity in its own right, with its own measuring procedure, i.e. in the same way as we are used to introduce electric charge. Then equation (1) takes over the role of a definition of the inertial mass, as the factor of proportionality between momentum and velocity.

[1] Pohl, R. W.: *Mechanik, Akustik und Wärmelehre*.– Springer-Verlag, Berlin, 1969.– S. 45

[2] Herrmann, F.: *The Karlsruhe Physics Course*, The Teacher's Manual, p. 23,
http://www.physikdidaktik.uni-karlsruhe.de/kpk/english/KPK_Teacher.pdf

[3] Herrmann, F., Schubart, M.: *Measuring momentum without the use of $p = mv$ in a demonstration experiment*, Am. J. Phyis. **57** (1989), p. 858

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