Subject:
To represent a field graphically one almost exclusively uses field line pictures.

Deficiencies:
Field lines are somewhat misleading when we ask the question of where a field is located. Since the field is distributed in space and the field strength changes from point to point, one might argue that this question does not make sense. But it does. When we ask about where is the air of the atmosphere of the earth, we know very well how to answer: by specifying the density distribution of the air. Qualitatively we might say: there is much air at the bottom, few at high altitude and almost none above 40 km.

The only quantity of a field that has the character of a density is the energy density. So, if we want to get an idea about where the field is, or how it is distributed in space, it is reasonable to ask for the energy density. However, the field line picture does not give an adequate information about the energy density, since we read it intuitively as a stream line diagram.

Consider an electrically charged sphere with radius $R$. The flux of the electric field strength far away, i.e. for a great distance $r$ from the center, is the same as for a small value of $r$. Thus, the field line picture suggests that in a volume element of thickness $dr$ far away out there is the same amount of field as in in element of the same thickness $dr$ further inside.

Such a conclusion would be correct for a radiating object like a star, if the energy density is taken as a measure of how much field there is: the energy is the same in every volume element of thickness $dr$.

For a static electric field however, such a conclusion would not be correct. The electric field strength decreases with the second power of $r$, the energy density with the forth power. Therefore, 90 % of the field energy is located within a sphere of radius 10 $R$, 99 % within a sphere of 100 $R$. One can say, that in this sense the field is located in a relative small region around the charged sphere.

We consider yet another example: the magnetic field of a solenoid. The field line picture suggests, that the field within the solenoid is more concentrated but that a considerable part of the “amount of field” is located outside of it. Again, the impression is very different when considering the energy distribution. If the solenoid is not too short, almost all of the energy is located within the solenoid – in the same way as almost the whole energy of the electric field of a capacitor is located between the plates of the capacitor.

Origin:
Usually, the field is defined as a region of space in which forces are acting. These forces can be recognized in the field line picture. As a consequence, the field lines are the only concrete anchor for an intuitive idea or a mental representation of the field.
Disposal:

Introduce the field as an autonomous system, i.e. not only as a mathematical tool for calculating forces. Since a field is an extended system we can represent it by a density distribution, even before showing that forces are exerted on a body which is brought into the field. Only thereafter we show that the “material” of the field is anisotropic. We proceed in the same way as we would do when explaining to somebody what we understand by the material “wood”. We would not begin by drawing lines which express the texture of the material. We rather begin by saying that wood is a homogeneous material with a certain density.

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