Subject:
“A measurement is the empirical determination of the actual value of a physical quantity.”

Deficiencies:
Measuring the value of a physical quantity is a standard task in physics. Measurements are carried out in order to find out or to verify the relation between physical quantities. When explaining why a measurement is necessary one often suggests the following idea: Before making the measurement the value is unknown, after the measurement it is known. Thus there are two states or situations: “not measured” and “measured”. Our citation is an extreme example for such a point of view. It says in addition that there exists an actual value. Sometimes it is stressed that we have to make a measurement because our senses are imprecise and unreliable.

This view is unfortunate in two respects.
First: It is not true that before executing a measurement nothing is known about the value of the physical quantity in question. And second: It is not the case that after the measurement we know the actual or exact value. Before making the measurement we know that the value is situated in a certain interval, which may be very large; after the measurement we also know that the value is in a certain interval, but this interval is smaller than that before the measurement. If by doing the measurement the interval has been strongly reduced, then it is a good measurement. If it is only slightly reduced the measurement is not so good.

Based on this observation we can define a number which characterizes the quality of a measurement of the quantity $X$: the ratio between the interval before and that after the measurement

$$\frac{X_{b2} - X_{b1}}{X_{a2} - X_{a1}}.$$

The index b refers to “before” and the index a to “after”. A more convenient definition would be the binary logarithm (lb) of this ratio

$$M = \text{lb} \left( \frac{X_{b2} - X_{b1}}{X_{a2} - X_{a1}} \right) \text{ bit}.$$ 

(1)

since it represents the information gain achieved by the measurement. It tells us by how many bits the information content of the value of a physical quantity has increased by the measurement. Suppose that before carrying out the measurement it is known that the value of the quantity under consideration is situated between 10 and 12, and after the measurement we know it to be between 10,6234 and 10,6236. We calculate

$$M = \text{lb} \left( \frac{12 - 10}{10,6236 - 10,6234} \right) \text{bit} = 13,3 \text{bit}.$$ 

The measuring instrument has provided 13,3 bit*. 

Historical burdens on physics
Origin:
In school physics it is common to classify a measurement as good if the
precision is better than about 5%. It is considered bad if the precision is
worse than 20 % more or less. This appraisement is rather arbitrary. Prob-a-
ibly it is due to the fact that the old pointer instruments had a measuring
precision of around a few percent. It may also be related to the fact that we
can determine the values of several quantities, like distances, velocities and
masses, by using our senses with a precision around 10 % to 50 %. The
idea might have been that an operation is called a measurement only if it
supplies values that are more precise than those which we get by using our
senses.

Disposal:
We recommend to take a measuring result seriously even if the precision is
in a range that usually is considered as imprecise. A “measurement” that is
realized with our senses is not necessarily a bad measurement, i.e. the in-
formation increase $M$ can be important.

*The definition is reasonable only as long as the uncertainty is small compared with the
measured value. However, it can be generalized in such a way that this case is also cov-
ered:

$$M = \log \left( \frac{X_{v2}}{X_{v1}} \right) \text{ bit}$$

Suppose it is known that the number of protons in the universe is between $10^{70}$ and $10^{90}$. Now somebody is able to show by means of some astrophysical measurement, that the
value is situated between $10^{75}$ and $10^{85}$. Our formula tells us, that the information gain is

$$M = \log \left( \frac{\log 10^{20}}{\log 10^{10}} \right) \text{ bit} = \log 2 \text{ bit} = 1 \text{ bit}$$

In the case that the precision is small in comparison with the measured value, the equation
simplifies to equation (1).

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