

## 4 Efficiency and Carnot factor

*Subject:*

The efficiency of a machine is defined by the quotient of the delivered useful energy to the total supplied energy:

$$\eta = \text{useful energy} / \text{supplied energy}$$

For a normal resistance heater, we obtain with this formula an efficiency of  $\eta = 1$ .

With a thermal engine one puts for the denominator of this expression all the energy which comes from the heat source and flows into the actual engine. If the thermal engine is ideal, i.e. if no entropy production takes place, the efficiency turns out to be the so-called Carnot factor:

$$\eta = (T_2 - T_1) / T_2$$

With a heat pump, one puts for the energy delivered in the desired form the energy which leaves the heat pump at the high temperature  $T_2$ , and obtains:

$$\eta = T_2 / (T_2 - T_1)$$

*Deficiencies:*

The efficiency is awkwardly defined. One expects from a reasonably defined efficiency that

1. its value lies between 0 and 1;
2. an ideal machine has an efficiency of 1;
3. a non-ideal machine has an efficiency  $< 1$ .

A machine is ideal when it works reversibly, or in other words: if no entropy is produced.

None of these three criteria is fulfilled by the definition of the efficiency indicated above. The efficiency of the heat pump is greater than 1, and the first condition is not met. The ideal, thus reversibly working Carnot machine has an efficiency which is less than 1, so the second condition is not met. The resistance heater, which works non-reversibly and is a notorious energy waster, has an efficiency of 1. Thus the third condition is not met.

*Origin:*

The search for the definition of an efficiency, an effectiveness or an economic coefficient of thermal machines accompanied for nearly a hundred years the intricate process of differentiating between energy and entropy. It is not found in Carnot's work. Carnot probably would not have appreciated the now common definition. We found it in the work of Helmholtz but we do not know for sure whether Helmholtz is the inventor of this measure.

Although it was an unfortunate choice from the beginning, it is at least understandable why such a definition was made. On the one hand heat pumps did not yet exist, i.e. machines that have, according to the above definition, an efficiency greater than 1. On the other hand, there were still no fuel cells, and it seemed that the only way to take profit of the energy of

carbon was to burn it. Therefore, it did not matter whether one attributed the low efficiency of a steam engine to the furnace or to the actual machine.

*Disposal:*

One uses the following definition for the efficiency:

$$\eta = P_{\text{ideal}} / P_{\text{real}}$$

$P_{\text{real}}$  is the energy consumption of the real machine, whose efficiency one would like to evaluate.  $P_{\text{ideal}}$  is the energy consumption of a machine or a plant that performs the same task, but works in a reversible way, i.e. without entropy production.

With this definition, one obtains  $\eta = 1$  for the reversibly working Carnot machine, because it is identical with the ideal machine of the same performance.

For the heat pump one always obtains a value of  $\eta$  which is smaller or equal to 1. If the machine works without any losses, that is, without friction losses, heat losses or losses in electric conductors, then it is ideal, and the efficiency will be equal to 1. In the case where we have such losses, will be less than 1.

A resistance heater supplies a certain entropy current (heat current)  $I_s$  at the high temperature  $T_2$ . The corresponding ideal machine is a heat pump, which supplies the same entropy current  $I_s$  at the same temperature  $T_2$ . It receives this entropy from the environment at temperature  $T_1$ . Thus, its energy consumption is

$$P_{\text{ideal}} = (T_2 - T_1) \cdot I_s$$

On the other hand the energy consumption of the resistance heater, which gives the same heat current  $T_2 \cdot I_s$ , is

$$P_{\text{real}} = T_2 \cdot I_s .$$

For the efficiency we obtain

$$\eta = P_{\text{ideal}}/P_{\text{real}} = (T_2 - T_1)/ T_2$$

i.e. is equal to the Carnot factor.

The resistance heater is wasting the more energy the higher the ambient temperature  $T_1$  is. Indeed, the higher  $T_1$ , the lower the energy expenditure for raising the entropy from ambient temperature to the desired value by means of the heat pump.

On the basis of the same consideration, for any other irreversible heater we obtain for the efficiency the Carnot factor, for example the furnace of a coal-fired power plant. Thus, the “weak point” in such a plant is not the nearly reversible turbine, but the irreversible working of the furnace.

The definition given here is known in thermodynamics as “second law efficiency”. It is introduced as an advanced concept. We propose to use from the beginning this definition, and call it simply “efficiency”.

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