

## Proceedings Sixteenth Marcel Grossmann Meeting 2021

1

### Teaching relativity: Computer-aided modeling

F. Herrmann and M. Pohlig\*

*Institute for Theoretical Solid State Physics, Karlsruhe Institute of Technology (KIT),  
Karlsruhe, Baden-Württemberg, Germany*

*\*E-mail: pohlig@kit.edu*

*www.kit.edu*

Mathematical derivations alone do not necessarily lead to physical understanding. Tools that can replace the mathematical treatment of a physical process and at the same time increase the physical understanding are computer-aided modeling programs, also called system dynamics software. Examples of such software are Stella, Berkeley Madonna, Wensim, Dynasys, Powersim or COACH 7. They solve differential equations and systems of differential equations with numerical methods. One works with a graphical user interface. We want to show how such a software can be used to get from a non-relativistic model to a relativistic model with only minimal modifications. Equating mass and energy alone, ensures that the model provides essential statements of relativistic dynamics: the existence of a terminal velocity for all physical motions, the relativistic dependence of the velocity of a body on its momentum, the relativistic relation between momentum and energy of a body.

*Keywords:* Teaching relativity, computer-aided modeling, system dynamic software, Coach 7.

### 1. Introduction

There are two reasons why learning relativity is difficult. First, there is a widespread belief that relativity is essentially a physics of reference frame changes. Second, mathematics, which seems to be indispensable for a first approach to relativity according to common usage, is a major, even often insurmountable, hurdle for many students. Therefore, the structure of our course follows two didactic recommendations: 1. avoid reference frame changes 2. reduce the use of mathematics to the most necessary. We fulfill the first recommendation by teaching dynamics before kinematics. Instead of postulating that the speed of light is invariant under reference frame changes, we declare from the very beginning that energy and mass are the same physical quantity. We call it energy when its value is measured in joules and mass when measured in kilograms. The second recommendation can be fulfilled if we use a suitable modeling software, a so-called system dynamics software (SDS). In a SDS, physical quantities and their relations are described by graphical symbols and are intuitively understandable. Since its usage is self-explanatory to a large extent, we do not need to go into details of the handling here.

As a representative of such a software we use COACH 7<sup>1</sup>.

## 2. Teaching Relativity using a system dynamics software

### 2.1. Momentum flow into a body

We create a model for a body which is accelerated from rest. Its momentum, which is zero at the beginning, increases as a constant momentum current flows into it (as a force is exerted on it). In a first step we treat this model non-relativistically. In the second step, the model is transformed into a relativistic model by a small modification.

In a SDS model, momentum  $p$  and other substance-like (extensive) quantities such as electric charge  $Q$ , energy  $E$ , or entropy  $S$  are represented by boxes. All these quantities change their values by an inflow or an outflow. (Entropy can also change its value by being produced). In the model, these currents are represented by thick arrows, Figure 1. Since in our lessons momentum is not introduced as a derived quantity, it gets its own unit of measurement, the Huygens, abbreviated Hy. The unit of its current is Huygens per second (Hy/s). The unit Hy is SI-compatible. We thus have  $1\text{Hy/s} = 1\text{ N}$ .

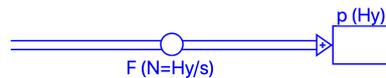


Fig. 1. A temporally constant momentum current  $F$  leads to a linear increase of the momentum  $p$ .

Since the body is accelerated from rest, its momentum at the beginning of its motion is  $0 \text{ Hy}$ . We assume the momentum current to be constant in time, we set e.g.  $F = 1 \text{ N}$ . When the model is started, the simulation begins running and the actual momentum is calculated for previously defined time steps.

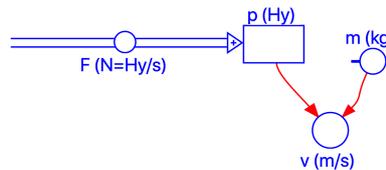


Fig. 2. The velocity is calculated from momentum and mass.

In order for our model to be able to tell us something about the velocity of the body, we must add the mass  $m$  of the body to the model, in Figure 2 represented by a circle. We choose  $m = 1 \text{ kg}$  for a first simulation and  $m = 2 \text{ kg}$  for the second one. The velocity is calculated using  $v = p/m$ . This relationship is stored in circular symbol for the velocity  $v$ . Thin, red arrows are pointing from the symbols for  $p$  and

$m$  to the symbol of  $v$ . This ensures that the actual values of  $p$  and  $m$  are available for the calculation of  $v$  at any instant of time during the simulation. In concrete terms, this means that while the simulation is running, the momentum is divided by the mass at fixed time steps, and thus the actual velocity  $v$  is calculated.

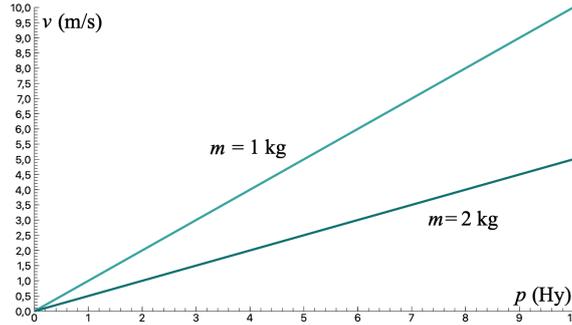


Fig. 3.  $v - p$  diagrams of bodies of masses 1kg and 2 kg.

The dependencies of the various variables occurring in the model can be displayed graphically in output windows. Figure 3 shows the  $v-p$  diagrams for two bodies of masses 1 kg and 2 kg.

### 2.2. Together with momentum energy is flowing into the body

We now add the kinetic energy of the body to our model, Figure 4..

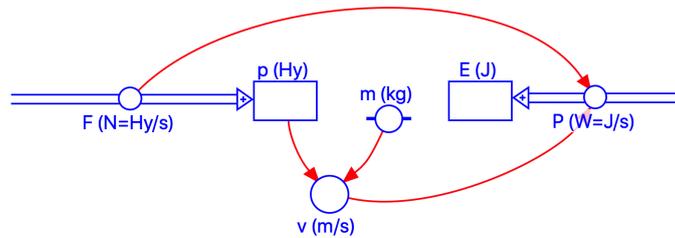


Fig. 4. The energy current  $P$  into the body is calculated from the velocity  $v$  of the body and the momentum current  $F$  (force).

Also the energy  $E$  is represented by a box. Since the body is accelerated from rest, not only the momentum but also the kinetic energy must have the initial value  $E_{\text{int}} = 0$ . Like the momentum current, the energy current (power) into the body is represented by a thick arrow. It can be calculated as  $P = vF$ . The software provides the energy-momentum diagrams for  $m = 1 \text{ kg}$  and  $m = 2 \text{ kg}$ , Figure 5. These graphs could be described by the formula

$$E_{\text{kin}} = \frac{p^2}{2m} . \tag{1}$$

However, this equation has not been used by the SDS.

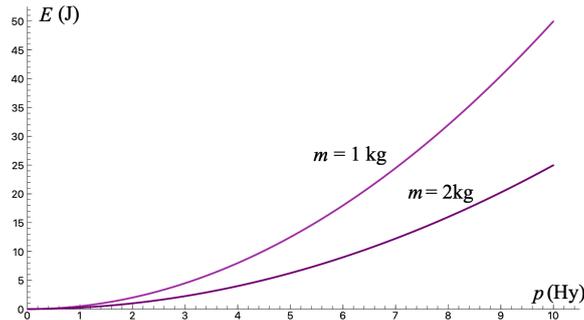


Fig. 5.  $E - p$  diagrams generated with the SDS for  $m = 1$  kg and  $m = 2$  kg.

### 2.3. From the classical to the relativistic model

The two models described up to now serve as basic models for others, such as for "free fall in the gravitational field of the earth", for "falling with friction" and others.

If momentum is replaced by other substance-like quantities such as electric charge or entropy, analogous models are obtained from electricity and thermodynamics, respectively. In this way, students learn to use the modeling software as a tool in other subfields of physics. So when they create relativistic models, they are already familiar with the tool. We now want to create a relativistic model by

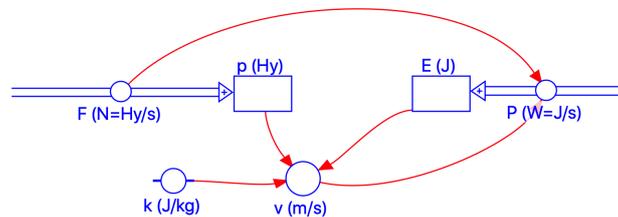


Fig. 6. Relativistic model: The constant mass is replaced by the total energy of the body.

modifying our non-relativistic model appropriately, Figure 4. For this purpose, only a small modification has to be made. The mass, which was originally a constant, is

now identified with the energy<sup>2</sup> according to

$$E = k \cdot m \quad (2)$$

Here, the energy is no longer the kinetic energy but the total energy of the body. Figure 6 shows the modified model. The initial value of the energy is no longer 0 J, but it is equal to the value of the energy that the body has at rest. We choose  $E_{\text{int}} = 10$  J and for another simulation  $E_{\text{int}} = 5$  J. To transform the unit kilograms into joules, we have to multiply the mass by  $k$ . We first set its value arbitrarily to  $k = 16$  J/kg. The actual true value of  $k$  will be discussed later. The velocity of the body is now calculated according to

$$v = \frac{p}{m} = \frac{kp}{E} \quad (3)$$

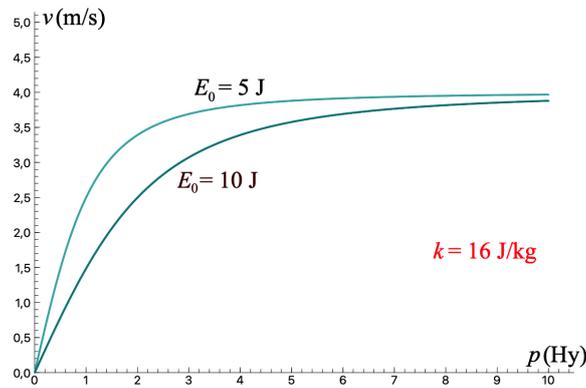


Fig. 7.  $v - p$  diagrams obtained with the relativistic model.

As Figure 7 shows, the  $v-p$  diagrams are different from those in Figures 3. It can be seen that the velocities of both bodies approach a common terminal velocity of 4 m/s. One "plays" with further, freely chosen values for the rest energy and always finds the same terminal velocity. Furthermore, the  $v-p$  diagrams show that light bodies reach the terminal velocity quicker than heavy ones. Finally, one recognizes that for sufficiently small momentum values the velocity of a body increases linearly with momentum. This confirms what is already known from non-relativistic mechanics.

The diagrams are graphs of the relation

$$v(p) = \frac{p}{m} = \frac{kp}{E} = \frac{kp}{\sqrt{E_0^2 + kp^2}} \quad (4)$$

However, we don't need to know this equation, or to derive it, in order to be able to read the important properties of this relationship from the diagrams.

The diagrams show that for relativistic motions the following rules hold:

- small momentum values:  $v \sim p$
- large momentum values:  $v = \sqrt{k}$

By choosing other initial values for the energy and other values for  $k$ , one easily gets convinced of the generality of these rules. For the diagrams in Figure 8, the values 9 J/kg, 16 J/kg and 25 J/kg were selected for  $k$ . The terminal velocity results to be 3 m/s, 4 m/s and 5 m/s, respectively.

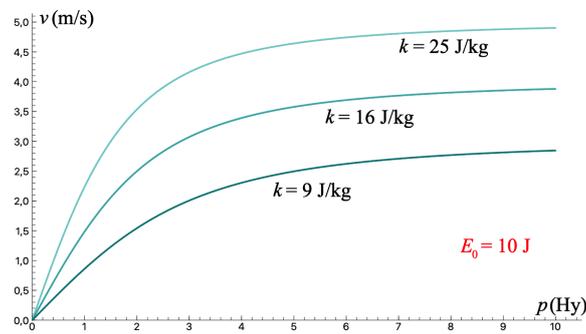


Fig. 8.  $v - p$  diagrams for different  $k$  values.

We see that the value of the terminal velocity is just the square root of  $k$ . While  $k$  used to be merely a conversion factor between kilogram and joule, it now acquires a physical meaning. Moreover, from the fact that a conversion factor for units is universal, the terminal velocity also has a universal value. It must be a universal constant. For this reason, we give the square root of  $k$ , i.e. the terminal velocity, a symbol of its own, namely

$$c := \sqrt{k} \quad (5)$$

Up to now, we had chosen the value of  $k$  and thus also that of  $c$  arbitrarily. But which value has nature given to  $k$  resp.  $c$ ? Experiments show that  $k = 9 \cdot 10^{16}$  J/kg and therefore  $\sqrt{k} = c = 3 \cdot 10^8$  m/s.

Another relationship that our model provides in the form of a diagram is that between energy and momentum, Figure 9. The diagrams shown are graphs of the equation:

$$E(p) = \sqrt{E_0^2 + kp^2} = \sqrt{E_0^2 + c^2p^2} \quad (6)$$

For small momentum values we get the non-relativistic relation between kinetic energy and momentum, shifted by the rest energy of the body:

$$E(p) = \frac{p^2}{2m} + E_0 \quad (7)$$

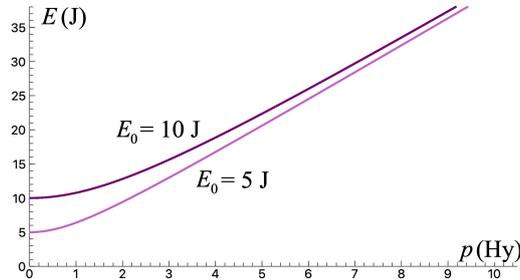


Fig. 9.  $E - p$  relation in the relativistic model. The rest energy was set to 5 J and in the second run to 10 J.

Moreover, we see that the ratio of energy and momentum is the same for large values of the momentum.

$$E(p) = \sqrt{k} \cdot p = c \cdot p \quad (8)$$

### 3. Conclusion

A non-relativistic SDS model which describes the behavior of a body whose momentum increases linearly, i.e. into which a constant momentum current is flowing, becomes a relativistic model by a small modification. The change consists in equating the quantities mass and energy. The new model then provides diagrams that are well-known from relativistic physics. The software provides them without using the respective equations. From these diagrams, important results of relativity can be interpreted and understood. The use of computer-aided modeling in teaching has the advantage of getting rid of the mathematical ballast and that one can concentrate on the physical content. Our experience shows that even younger students can easily learn a software like COACH 7 and use it to work out challenging results in relativistic dynamics.

### References

1. Coach 7 for Desktop and Tablet: CMA (Centre for Microcomputer Application) [https://cma-science.nl/downloads\\_en](https://cma-science.nl/downloads_en) accessed July 26 2021
2. Herrmann, F. et al., *The Karlsruhe Physics Course: Mechanics: for the upper secondary school*, (2019) [http://www.physikdidaktik.uni-karlsruhe.de/index\\_en.html](http://www.physikdidaktik.uni-karlsruhe.de/index_en.html) accessed August 2021.