

Statics in the momentum current picture

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(Received 21 December 1982; accepted for publication 3 May 1983)

Newton's Second Law is equivalent to the continuity equation for momentum in integral form. This insight leads to an alternative picture of forces as momentum currents. The purpose of the present paper is to introduce a new approach to statics problems and their solutions in terms of momentum currents. In particular, the relation between the distribution of momentum currents and the elastic stresses within a medium will be considered and a simple way to pictorially represent those stresses with the help of momentum flow diagrams will be discussed. Handling statics problems in the momentum current picture immediately displays their relationship to analogous problems in electrical network theory. This uncovers a structural relationship between the role of electric charge in the theory of electricity and the role of momentum in mechanics. For example, it will be shown that the familiar method for the solution of statics problems in terms of free-body diagrams is equivalent to the use of a junction rule for momentum currents.

I. INTRODUCTION

Every force can be visualized as a momentum current, i.e., "force" is just another name for a momentum current.^{1,2} To see this clearly, imagine two wagons accelerating toward one another with the help of a motor which winds up the rope connecting them together (Fig. 1). The momentum of the left wagon increases, that of the right wagon decreases. Instead of saying the wagons are exerting forces upon each other by way of the rope, one can say that a momentum current is flowing from the wagon at the right through the rope to the wagon at the left.

This is more than just a figure of speech. The concept of momentum current underlies an entirely different approach to mechanics than the concept of force allows: If the amount of momentum contained within a body or a region

of space changes, a net momentum current must flow through the boundary surface of the body or the region of space. Accordingly, in this picture, momentum conservation is understood locally. The result is what could be

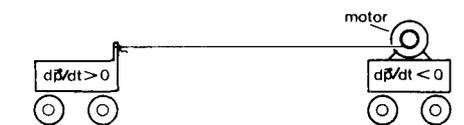


Fig. 1. Two wagons accelerating toward one another with the help of a motor which winds up a rope connecting them together. The momentum of the left wagon increases, that of the right wagon decreases showing that momentum \mathbf{p} is flowing from the wagon at the right through the rope to the wagon at the left.

called a *local-causes* approach to mechanics, i.e., a mechanics without action-at-a-distance. Furthermore, from the point of view of momentum currents, new kinds of questions come up which seem awkward in the force picture. For example, when one says: "A momentum current flows from body 1 to body 2," the question immediately comes up "By which path does this current flow?" The analogous question in the force picture: "What is the distribution of stresses around and between the bodies?" sounds complicated especially, for example, if the two bodies are interacting via a field.

The purpose of this paper is to present an alternative approach to the statics of structures in terms of momentum currents. In particular, the relation between the distribution of momentum currents and the elastic stresses within a medium will be discussed and a simple way to pictorially represent these stresses with the help of momentum flow diagrams will be presented.

Statics problems are traditionally solved in terms of forces with the help of free-body diagrams which show all the forces acting at each junction or knot in the structure. We will show that the rules for constructing such free-body diagrams are identical with a "Kirchhoff's First Law" for momentum currents.

The general problem of the statics of structures will be treated by way of two typical examples. The first and easier example is a lantern suspended by several cables. The second, somewhat more complicated example, is the beam of a crane.

II. MOMENTUM CURRENTS IN STATIC STRUCTURES

Newton's Second Law is traditionally written as

$$\frac{d}{dt} \mathbf{p} = \sum_{i=1}^N \mathbf{F}(i), \quad (1)$$

where \mathbf{p} is the momentum of, say, a body and $\mathbf{F}(i)$ is the i th of N forces acting on the body. In the momentum current picture, Newton's Second Law is seen to be equivalent to the integral form of the continuity equation for momentum

$$\frac{d}{dt} \mathbf{p} + \sum_{i=1}^N \mathbf{I}(i) = 0. \quad (2)$$

Here, $\mathbf{I}(i)$ is the net momentum current flowing out of, say, a body through the i th channel. Expressed verbally, (2) reads:

"The value of the momentum \mathbf{p} contained within an arbitrary region of space R can change in time only if a net momentum current $\sum_{i=1}^N \mathbf{I}(i)$ flows through the N channels penetrating the (closed) boundary surface of R ."

A comparison of (2) with (1) shows that every force acting on an object is equivalent to a momentum current flowing through a channel, say, a rope, a bar, or even a field into the object, i.e., $\mathbf{F}(i) = -\mathbf{I}(i)$.

A static structure is characterized by the fact that the first term in (2) is zero for every region R within the structure. Accordingly, (2) becomes

$$\sum_{i=1}^N \mathbf{I}(i) = 0. \quad (3)$$

Whenever a momentum current is flowing through a material medium, the medium is being deformed. This deforma-

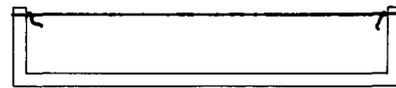


Fig. 2. A static arrangement consisting of a rubber band stretched across a solid frame. The distortion in the rubber band shows that momentum is flowing throughout the structure. No dissipation of energy accompanies this flow of momentum showing that the flow of momentum is a "super current."

tion can be used to measure such a current. Indeed, this is just what a spring balance does.

The flow of a momentum current has nothing necessarily to do with the motion of the channel through which it flows. For example, momentum is flowing through the rubber band stretched across the static arrangement shown in Fig. 2. Furthermore, since there is no dissipation of energy accompanying the flow of momentum in such a static arrangement, e.g., since the rubber band does not heat up, the momentum currents flowing through a static structure are super currents, analogous to electric super currents.

III. A JUNCTION THEOREM FOR MOMENTUM CURRENTS

A standard statics problem is that of a lantern suspended by several cables (Fig. 3). The weight of the lantern is given and the problem is to determine the magnitude and direction of the forces acting in the cables 1 and 2. The solution is obtained by constructing a free-body diagram for the junction where the cables are attached above the lantern. The weight $\mathbf{F}(3)$ of the lantern, i.e., the force of gravity acting on the lantern, must be compensated for by the forces $\mathbf{F}(1)$ and $\mathbf{F}(2)$ acting in cables 1 and 2, respectively, at the junction:

$$\mathbf{F}(1) + \mathbf{F}(2) = -\mathbf{F}(3)$$

or

$$\sum_{i=1}^3 \mathbf{F}(i) = 0. \quad (4)$$

One is able to use a free-body diagram as an aid to solving this problem because the force vectors $\mathbf{F}(1)$ and $\mathbf{F}(2)$ lie along the cables 1 and 2, respectively.

The method of solution of this problem in the momentum current picture is identical to that above although the interpretation of the solution is quite different: The free-body diagram is a manifestation of a "Kirchhoff's First Law" for momentum currents. To see this more clearly, consider the analogous situation from the theory of elec-

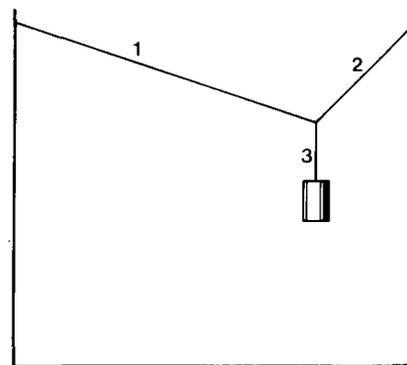


Fig. 3. Classical statics problem of a lantern suspended by three cables.

tricity. If several, say, N wires converge at a single location (junction) within an electrical network, the following statement is known to hold true: At any junction the algebraic sum of the electric currents is zero. This is Kirchhoff's First Law (sometimes called Kirchhoff's Junction Theorem). This law is usually written in the form

$$\sum_{i=1}^N I_Q(i) = 0, \quad (5)$$

where $I_Q(i)$ is the electric current flowing through the i th channel (wire) of the circuit.

Kirchhoff's First Law is a special case of the continuity equation for electric charge Q

$$\frac{d}{dt}Q + \sum_{i=1}^N I_Q(i) = 0. \quad (6)$$

This equation is a consequence of the conservation of charge. If there are no sources or sinks of electric charge in a region containing the junction, the value Q of the electric charge in this region remains constant in time and (6) tells us, that

$$\sum_{i=1}^N I_Q(i) = 0.$$

Equation (3) is valid for static arrangements and follows from the general continuity equation (2) for momentum in the same way that (5) follows from (6). It is therefore suggestive to call (3) the "Junction Theorem for Momentum Currents." We understand a mechanical junction thereby to be a location at which several momentum currents converge, for example, the location above the lantern in Fig. 3 where the three cables converge. The junction theorem for electric currents states that a sum of scalar quantities equals zero, whereas the same law for momentum currents makes a similar statement about a vector sum.

For a cable or a beam under tension or compression along its length, the momentum current vector is parallel to the cable or beam. The following rule can be shown to hold for the orientation of the vector \mathbf{I} :

The vector $\mathbf{I}(i)$ points toward a junction for a momentum current through a cable or beam (i) under tension and away from a junction for a momentum current through a beam (i) under compression.

Let us now return to the lantern problem in the momentum current picture. According to the above rule, each of the three vector currents $\mathbf{I}(i)$, $i = 1, 2, 3$, points toward the junction. The mass m of the lantern is supposed to be

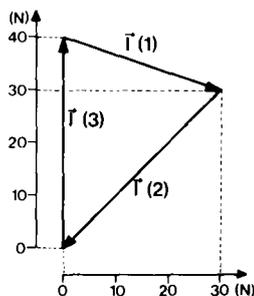


Fig. 4. Application of Kirchhoff's Junction Theorem to the junction above the lantern in Fig. 3. The figure shows the vector addition of the three momentum currents $\mathbf{I}(i)$, $i = 1, 2, 3$ converging at the junction. The value of $|\mathbf{I}(3)|$ is taken to be 40 N.

known and, therefore, the magnitude of $\mathbf{I}(3)$ (recall Fig. 3) is given by

$$|\mathbf{I}(3)| = mg, \quad (7)$$

where g is the acceleration due to gravity (see the comment at the end of Sec. V). Thus, given the angles between cables 1, 2, and 3, the momentum current vectors $\mathbf{I}(1)$ and $\mathbf{I}(2)$ can be determined with the help of the Junction Theorem for Momentum Currents (Fig. 4).

IV. THE DIRECTION OF FLOW OF MOMENTUM CURRENTS

Since momentum is a vector it follows from (2) that the momentum current $\mathbf{I}(i)$ is also a vector. The vector momentum current $\mathbf{I}(i)$ can be described in terms of a (symmetric) tensor, the momentum current density $\mathbf{j}(\mathbf{r})$:

$$\mathbf{I}(i) = \int_{S_i} d\sigma \mathbf{j}(\mathbf{r}). \quad (8)$$

The surface integral in (8) extends over the surface S_i cutting the channel i . The quantity $d\sigma$ appearing in (8) is a unit surface element lying along the channel and directed away from the junction under consideration. The momentum current density tensor $\mathbf{j}(\mathbf{r})$ is commonly known as the (negative) stress tensor.

Just as the electric current I_Q of the electric charge Q is represented by a surface integral over the charge current density $\mathbf{j}_Q(\mathbf{r})$, the vector momentum current \mathbf{I} of the momentum \mathbf{p} is represented by a set of three "scalar" momentum currents I_x , I_y , and I_z of the three "scalar" quantities: the x component p_x , the y component p_y , and the z component p_z of the momentum \mathbf{p} :

$$I_x = \int_{S_i} d\sigma \mathbf{j}_x(\mathbf{r}) \quad (9)$$

and similarly for I_y and I_z [where the channel label (i) has been dropped for simplicity]. We call p_x , p_y , and p_z , "x momentum," "y momentum," and "z momentum," for short. In the same way, we call \mathbf{j}_x , \mathbf{j}_y , and \mathbf{j}_z the "x-momentum," "y-momentum," and "z-momentum current density," respectively.

The momentum density "vectors" \mathbf{j}_x , \mathbf{j}_y , and \mathbf{j}_z are obtained from $\mathbf{j}(\mathbf{r})$ by projecting it onto three independent directions, say, the x , y , and z directions of an arbitrarily chosen set of Cartesian coordinate axes:

$$\mathbf{j}_x(\mathbf{r}) = \mathbf{j}(\mathbf{r}) \cdot \mathbf{x}_0 \quad (10)$$

and similarly for $\mathbf{j}_y(\mathbf{r})$ and $\mathbf{j}_z(\mathbf{r})$. Here \mathbf{x}_0 represents a (constant) unit vector lying along the x axis. If the projections of the momentum current density tensor $\mathbf{j}(\mathbf{r})$ are known in three independent directions, then the stress tensor is completely determined and, consequently, the stress state of the corresponding physical system is fully specified.

The advantage of dealing with the three independent "vector" fields \mathbf{j}_x , \mathbf{j}_y , and \mathbf{j}_z is that these can be pictorially represented in the usual manner in terms of field lines. The field lines of, say, \mathbf{j}_x , show how x momentum flows through the object being considered.

When speaking of the direction of flow of a physical quantity one generally means the sense of direction along the corresponding current density vector. Since there are three current density vectors associated with the flow of a vector quantity like momentum, it does not make sense to

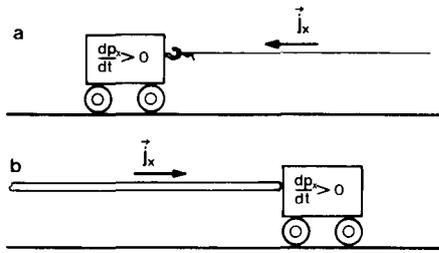


Fig. 5. Examples used to illustrate the direction of momentum flow. (a) A body is being accelerated with the help of a string by pulling it to the right. Positive x momentum is flowing into the body from right to left, i.e., in the negative x direction through the string. The string is under tension. (b) A body is being accelerated with the help of a rod by pushing it to the right. Positive x momentum is flowing into the body from left to right, i.e., in the positive x direction through the rod. The rod is under compression.

speak about the direction of flow of momentum as such. Rather, one must refer to the direction of flow of each component p_x , p_y , and p_z of the momentum.

In static structures made up entirely of cables and beams, the direction of flow of each component p_x , p_y , and p_z is necessarily parallel to the axis of each cable and beam, exactly in the same way that the direction of flow of electric charge is necessarily parallel to the axis of each wire in a network. However, it frequently occurs that the direction of flow of, say, the x momentum within a cable or beam is opposite to that of, say, the y momentum. We will come back to this point below.

The relationship between the sense of direction of the field lines of the momentum current density \mathbf{j}_x , \mathbf{j}_y , and \mathbf{j}_z and the state of stress within an elastic medium can be stated in terms of a single rule. The derivation of this rule is easy to illustrate with the help of a few simple examples. The body in Fig. 5(a) is being accelerated by pulling it to the right. Accordingly, its positive x momentum is increasing. Thus, positive x momentum is flowing into the body from right to left, i.e., in the negative x direction, through the channel (a string) under tension. In the same way, in Fig. 5(b), (positive) x momentum is flowing into a body from left to right, i.e., in the positive x direction. Here the channel (a rod) used to accelerate the object to the right is under compression. Analogous statements hold for the acceleration of a body in the y or z directions. Summing up: the field lines of the x -momentum current density are oriented in the negative (positive) x direction within a channel under tension (compression) lying along the x axis. The same is true when speaking about the field lines of y or z momentum.

If the momentum conducting channel does not lie parallel to a coordinate axis, the field lines of at least two components of the momentum are nonvanishing within the channel (Fig. 6). In Fig. 6(a) and 6(b), both the x and the y momentum of the body increase. In Fig. 6(a), both x and y momentum flow into the body through an expanded string from the upper right to the lower left of the figure, i.e., both the x and y components of \mathbf{j}_x and \mathbf{j}_y are negative. In Fig. 6(b) both x and y momentum flow into the body through a compressed rod from the lower left to the upper right of the figure, i.e., both the x and y components of \mathbf{j}_x and \mathbf{j}_y are positive.

In Fig. 6(c) and 6(d), the x momentum of the body decreases and the y momentum of the body increases. In Fig. 6(c), both x and y momentum flow through an expanded string: x momentum flows out of the body from the lower

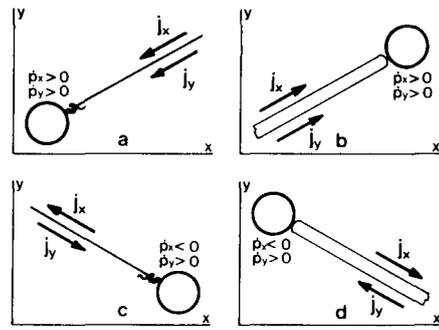


Fig. 6. Further examples used to illustrate the direction of momentum flow (see text).

right to the upper left of the figure and y momentum flows into the body from the upper left to the lower right, i.e., both the x component of \mathbf{j}_x and the y component of \mathbf{j}_y are negative. In Fig. 6(d), both x and y momentum flow through a compressed rod: x momentum flows out of the body from the upper left to the lower right of the figure and y momentum flows into the body from the lower right to the upper left, i.e., both the x component of \mathbf{j}_x and the y component of \mathbf{j}_y are positive.

Figure 6(a) and 6(b) illustrates examples whereby both x and y momentum flow in the same direction. Figure 6(c) and 6(d) illustrates examples whereby x and y momentum flow in opposite directions. Nevertheless in all four cases, the following rule applies:

The x (y , or z) component of \mathbf{j}_x (\mathbf{j}_y or \mathbf{j}_z) is *negative* within a channel under pure *tension* and is *positive* within a channel under pure *compression*.

It is easy to see from the above examples how it can be that, even in a fully symmetric configuration like the one shown in Fig. 2, a particular direction is favored as the direction of flow along the field lines of the x -, y -, or z -momentum current density: The asymmetry is a consequence of having selected a particular direction as the positive x , y , or z axis.

V. MOMENTUM FLOW DIAGRAMS

The Junction Theorem given in Sec. III actually solves the lantern problem as it was originally posed. However, the treatment of problems in the momentum current picture brings up the additional question: "How does momentum flow in the structure?" This question can be answered in more or less detail as desired.

A complete answer requires the specification of the momentum current density $\mathbf{j}(\mathbf{r})$ throughout the structure. As explained in Sec. IV, the tensor field $\mathbf{j}(\mathbf{r})$ can be pictorially represented in terms of the x -, y -, and z -momentum current densities $\mathbf{j}_x(\mathbf{r})$, $\mathbf{j}_y(\mathbf{r})$, and $\mathbf{j}_z(\mathbf{r})$. Thus, the specification of the field lines for the latter determines $\mathbf{j}(\mathbf{r})$ and, consequently, leads to a complete answer to the above question.

In many cases, however, one is satisfied with a less exhaustive answer. Often, for example, momentum can flow only in supports, rods, cables, struts, beams, and the like throughout the structure. In such cases, one is only interested in the specific channels, through which each of the three "kinds" of momentum flow and how much of each kind of momentum flows through any cross section along its way. The solution of this problem can be conveniently

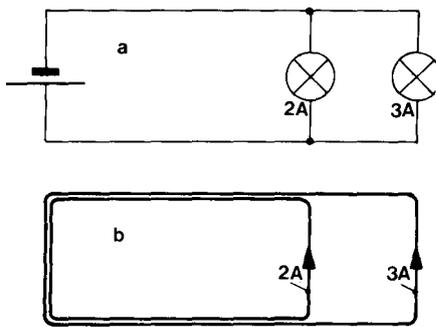


Fig. 7. (a) An electrical network consisting of a battery and two light bulbs. (b) One possible and convenient way to illustrate the flow of charge through the network of Fig. 7 (a).

presented in a pictorial fashion similar to that illustrated in Fig. 7 for the example of an electric network.

We will call diagrams of the type shown in Fig. 7(b), "flow diagrams." In such diagrams, the flow throughout a network is decomposed into several simple current loops which overlap one another. Each of these simple current loops is indicated by a single closed line which actually represents a bundle of current density field lines. The direction of flow along these field lines is indicated by one or more arrow heads sketched on the single representative line. The magnitude of flow (or current strength) usually indicated by the density of the field line bundle times the

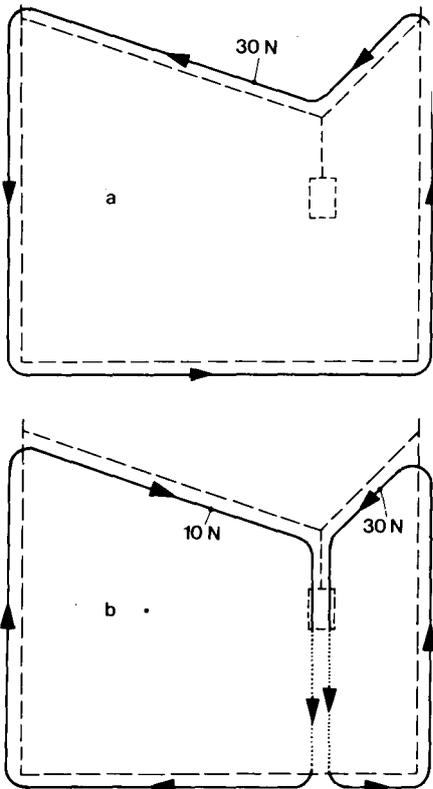


Fig. 8. Momentum flow diagram for the lantern problem of Fig. 3. (a) Flow diagram for the flow of j_x throughout the structure. (b) Flow diagram for the flow of j_z throughout the structure. The flow of momentum through the gravitational field is suggested by the dotted line. Details of the flow of momentum through the gravitational field will be discussed in a later publication.

area of the channel is now simply given numerically next to each representative line.

In order to represent the flow of momentum throughout a "mechanical network," three such flow diagrams must be sketched, one for each of the three components of momentum p_x , p_y , and p_z . In the case of the lantern problem and taking all supports to lie in the xz plane, only the x and the z components of momentum are of interest, i.e., only the x - and z -momentum current densities are nonzero. Accordingly, two momentum flow diagrams can be sketched for this problem (Fig. 8). The current strengths I_x or I_z associated with the flow of x and z momentum, respectively, can be obtained, for example, from Fig. 4: The strength of the x -momentum current in each of the three cables 1, 2, or 3 is equal to the magnitude of the x component $I_x(1)$, $I_x(2)$, $I_x(3)$ of the vector $I(1)$, $I(2)$, or $I(3)$, respectively. An analogous statement holds for the strength of the z -momentum current in each cable. The orientation of flow in each case is obtained from the rule in Sec. IV.

It is obvious from Fig. 8(b) that z momentum continues to flow from the lantern through the gravitational field to the earth. This has been suggested by the two dotted lines in the figure. The exact distribution of stresses within the gravitational field follows directly from a metric theory³ of gravity and can be shown in the weak-field limit to have the same form (with opposite overall sign) as the familiar Maxwell's Stress Tensor for a pure electric or magnetic field.⁴ Details of the flow of momentum through the gravitational field will be discussed in a later publication dealing with momentum currents in fields.

VI. MOMENTUM CURRENTS IN THE BEAM OF A CRANE

A somewhat more complicated example of a static structure is the beam of a crane [Fig. 9(a)]. In the usual presentation of this problem, the mass m hanging from the end of the beam is given and one is asked to calculate the forces in the various supports under the assumption that there is no

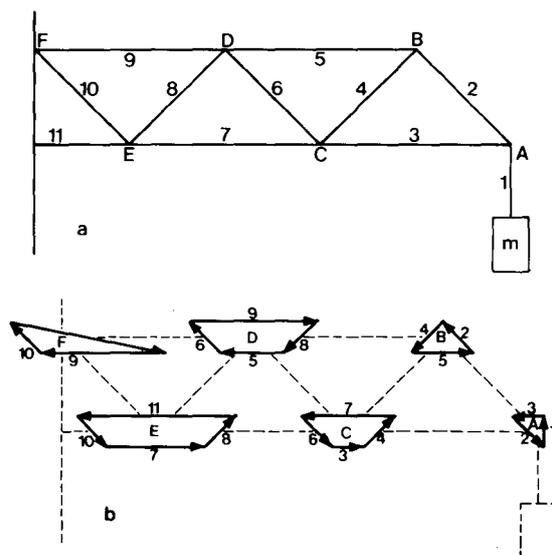


Fig. 9. (a) Illustration of the beam of a crane with an object of mass m suspended from the beam by a cable labeled 1. The individual supports of the beam are labeled 2-11 and the junctions are labeled A-F. (b) Momentum current polygons sketched at each junction of the beam in Fig. 9 (a).

bending, i.e., each beam member is under compression or tension only. The solution to this problem is arrived at by repeated applications of a free-body diagram at each junction.

In the momentum current picture, the solution is again formally identical with the traditional one. Only the interpretation is different. The solution, i.e., the magnitude of the momentum currents $I(i)$ in each of the supports $i = 1, 2, \dots, 11$, is obtained by successive application of the Junction Theorem for Momentum Currents. A momentum current triangle or quadrangle is constructed at each junction beginning at junction A and proceeding from right to left from one junction to the next.

The construction is illustrated in Fig. 9(b). The number (i) on each momentum current vector $I(i)$ labels the support in which the current flows. Each number (except 11) participates in two momentum current polygons since each support has two junctions. The two momentum current vectors of each pair have opposite directions since the junction to which each member of the pair are referred are at opposite ends of the corresponding support.

Let us now consider the momentum flow diagrams. We begin with the z -momentum current. We know that the z component of each \mathbf{j}_z field line is oriented in the ($-$) z direction in every support under tension and in the ($+$) z direction in every support under compression. This means that the \mathbf{j}_z field lines point vertically downward from junction A in Fig. 9(a) through the cable labeled 1 and into the suspended mass. These field lines must flow into the junction along one or both of the supports 2 or 3. Since I_z and, thus, \mathbf{j}_z vanish in all the horizontal supports, the \mathbf{j}_z field lines can only be sketched in the diagonal support 2. Continuing this line of reasoning successively at junctions B, C, D, E, and F, we arrive at the flow diagram shown in Fig. 10(a). Since there are no forks in the \mathbf{j}_z field lines, the z -momentum current must have the same magnitude in all the diagonal supports.

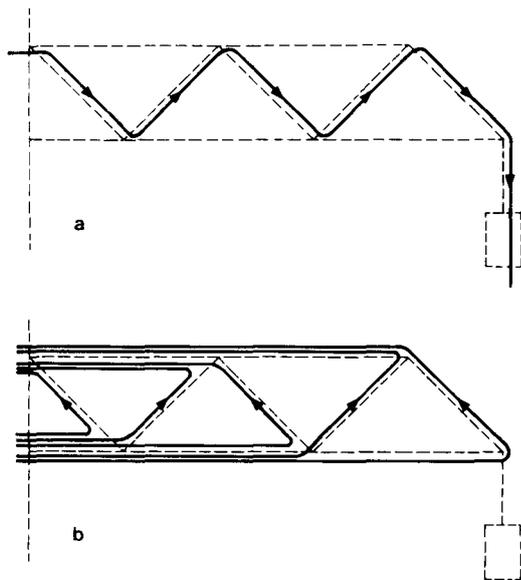


Fig. 10. Flow diagrams for the flow of momentum throughout the beam of Fig. 9. Because of the angles involved, namely 45° , the same current values are associated with all flow lines. (a) Flow of z momentum. (b) Flow of x momentum.

Referring back to Fig. 9(b) and applying the rule from Sec. III above, the state of stress (tension or compression) can be read from the direction of the current vectors $I(i)$ at each junction (i). In particular, one sees that the upper horizontal supports are all under tension whereas the lower horizontal supports are all under compression. Accordingly the field lines of the x -momentum current density are oriented in the ($-$) x direction in the upper supports and in the ($+$) x direction in the lower supports. Still remaining to be determined for a full description of the momentum flow throughout the beam is the specification of the x -momentum current in the diagonal supports. The direction of flow can be inferred from the rule of Sec. IV, the magnitude of the x component of the momentum current vector can be read from the momentum current polygons in Fig. 9(b). The complete result for the flow of x momentum throughout the beam is given by the flow diagram in Fig. 10(b).

The x momentum flows throughout the beam in a series of current loops, each of which is closed in the overall support of the crane at the left of the beam. The x -momentum currents are "induced" throughout the beam so that the z -momentum currents caused by the hanging object can flow in the x direction from the support at the left to the weight hanging at the right end of the beam.

VII. EXTENSION TO PROBLEMS WITH TORQUE

Although only a certain class of static problems have been treated here, the extension of these ideas to general problems is straightforward. In such cases, an additional set of conditions analogous to (3) and of the form

$$\sum_{i=1}^N \mathbf{I}_L(i) = 0 \quad (11)$$

must be considered. In (11), the symbol $\mathbf{I}_L(i)$ refers to the flow of angular momentum through channel (i). Equation (11) is actually a "Junction Theorem for Angular Momentum Currents." Traditionally, an angular momentum current is referred to as a torque. The conceptual treatment of angular momentum currents is given in detail elsewhere.⁵

VIII. BENDING BEAMS

The beam of a crane represents a simplified model of a solid, homogeneous beam. Accordingly, the stresses within a solid beam supporting a heavy object can be understood in a way similar to that by which the stresses within the beam of a crane were understood above. Once again, z mo-

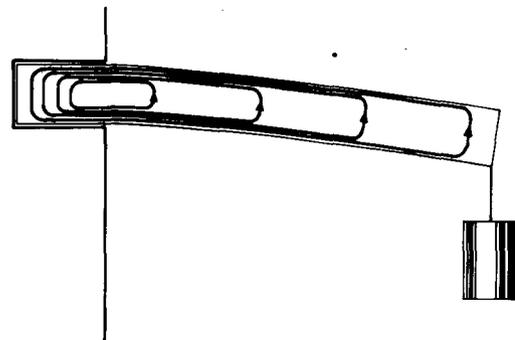


Fig. 11. Illustration of the x -momentum current density within a solid, homogeneous beam supporting a weight at one end.

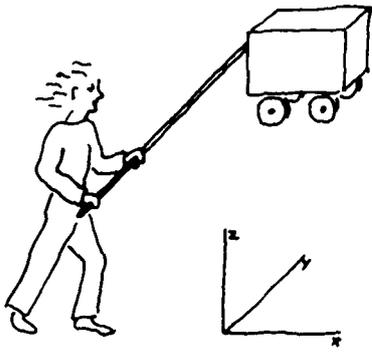


Fig. 12. Example to illustrate the direction of the momentum flow through a bending rod: A wagon is being accelerated to the right by sweeping it along with a rod from the side.

momentum must flow horizontally from the support at the left of the beam suspended weight and, as before, this is only possible if x -momentum current loops are induced throughout the beam (Fig. 11). The result is that the upper side of the beam is under tension and the lower side of the beam is under compression. These stresses increase in magnitude proceeding from right to left within the beam. The j_x field lines running perpendicular to the x axis correspond to torque in the beam. This is easy to understand from the example in Fig. 12. There, a wagon is being accelerated to the right by sweeping it along with a rod from the side. The x -momentum flows perpendicular to the x axis in the positive y direction through the rod and into the wagon. This flow of momentum is evidenced by bending in the rod.

IX. CONCLUSIONS

The purpose of this paper is to present an alternative approach to statics problems and their solutions with the help of momentum currents.

Handling statics problems in the momentum current picture immediately displays their relationship to analogous problems in electrical network theory. This uncovers a structural relationship between the role of electric charge in the theory of electricity and the role of momentum in a local-causes approach to mechanics. For example, it has been shown that the familiar method for the solution of statics problems in terms of free-body diagrams is equivalent to the use of a junction rule for momentum currents.

Another advantage of the momentum current picture is that it immediately leads to the question of the paths along which momentum is flowing, i.e., to the question of the momentum current density distribution, within the considered object. This question opens up the way to a complete solution of the elastic properties of the considered object in terms of the momentum current density tensor. However, the momentum current picture also allows for a less exhaustive solution in terms of momentum flow diagrams.

An approach to statics with the help of momentum currents provides a simple, pictorial representation of the distribution of stresses within a continuous medium, for example, within a macroscopic body or even the electromagnetic field,⁴ an insight which is otherwise very difficult to gain from the usual (highly mathematical) approaches to the theory of elasticity.

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